C.R. Rao at 100

October 2, 2020

Terry Speed had the tough task of outlining a very long—and extraordinarily productive—life, to mark C.R. Rao's 100th birthday:

Professor Calyampudi Radhakrishna Rao (hereafter CRR) was born on September 10, 1920 in Hadagali, Karnataka State, India. He received an M.A. in mathematics in 1940 from Andhra University and an M.A. in statistics in 1943 from Calcutta University. From Cambridge University, he received a PhD in 1948, completing a thesis entitled *"Statistical Problems of Biological Classification"* under the statistician and geneticist R.A. Fisher.

CRR's research career was truly remarkable. He had no knowledge of statistics prior to joining the Indian Statistical Institute (ISI) in 1941 for a one-year training course, yet within a few months he was publishing research in experimental design jointly with K.R. Nair. At about this time, Calcutta University started a master's degree program in statistics, and CRR transferred to that, completing it two years later in mid-1943. By then he had well established research interests in combinatorial design, linear statistical estimation, multivariate statistical analysis and statistical characterization problems. These were to remain the broad themes of his lifelong research. The design research in his M.A. thesis led to his landmark 1947 paper introducing orthogonal arrays, which are widely-used combinatorial entities finding applications in the statistical design of experiments, coding theory, cryptography and software testing. The linear model research in this thesis took off from contributions of his teacher R.C. Bose, while multivariate analysis research extended methods CRR learned from S.N. Roy. Some was published soon after presenting the thesis, while much more was further developed over the next three years. The last part of this outstanding M.A. thesis published in 1947 solved a characterization problem involving the linearity of regressions posed in econometrics. It foreshadowed a lifelong interest in both topics. CRR was a co-founder of the Indian Econometric Society, and in 1972, with two top Russian mathematicians, he co-authored a pioneering book on characterization problems in statistics.

Following the completion of his outstanding M.A. degree, CRR was appointed by P.C. Mahalanobis to a position as technical apprentice in the Indian Statistical Institute (ISI). His duties were to assist Mahalanobis in the editorial work of *Sankhyā, the Indian Journal of Statistics*, and to take charge of the data analysis for an anthropometric project involving 9–11 measurements on about 4,000 individuals from the Indian state of Uttar Pradesh collected during the 1941 census. He also lectured part-time at Calcutta University to the master's class he had recently left. CRR had been unusually productive while completing his M.A., but in the period between then and his departure for Cambridge in August 1946 he was extraordinarily so. In that two-and-a-half years, he established himself as a world leader in statistics. His most famous publication from that time is a 1945 paper in which he derives a lower bound on the variance of an unbiased estimator of an unknown parameter. This result came to be known as the Cramér–Rao inequality, as it was derived independently by H. Cramér in 1946, and also M. Fréchet in 1943 and G. Darmois in 1945. The result is entirely analogous to the famous 1927 quantum mechanical uncertainty principle of W. Heisenberg and is derived in a similar manner. This paper also contains an important application of Fisher's sufficiency, now known as Rao–Blackwellization, as the idea was independently discovered by D. Blackwell in 1947, and the introduction of differential geometric notions into parametric statistical inference, a development that subsequently grew into a very active subfield of statistics. His geodesic distance between populations (distributions) extended those of Mahalanobis and Bhattacharya, and is used today in ecology. A noteworthy short paper from this period by CRR and his ISI colleague S.J. Poti gave the first example of a score test to test a null-hypothesis with a one-sided alternative. A couple of years later, during his PhD research with Fisher, CRR met a testing problem in genetics for which the score test provided an appropriate solution, and developed a general large sample theory for such tests. In 1948 he published this

theory, and now it is recognized as the important closing of a triangle in the likelihood graph, the other two sides of which are J. Neyman and E.S. Pearson's 1933 likelihood ratio tests, and the 1943 tests of A. Wald. Score tests have had an enormous impact in statistics, especially in econometrics. Other substantial research contributions in this period include further work on a general theory of least squares and tests with discriminant functions in multivariate analysis. The latter was motivated by CRR's desire to develop further the ideas of Fisher and Mahalanobis, especially as they related to his ongoing anthropometric studies, which were not to be completed and published until 1949, after his return from Cambridge to Calcutta.

In 1946 Mahalanobis received a request from J. Trevor of the Anthropology Department of Cambridge University, to send someone who could use methods of multivariate analysis developed at the ISI to analyse measurements made on skeletal material dug out from ancient graves in Africa, to trace the origin of the people who lived in the region. CRR was deputed to work on the project and spent the years 1946–48 in Cambridge as a Research Scholar at the University Museum of Archaeology and Ethnology. He produced a report on the statistical analysis of the skeletal measurements, which was incorporated in his thesis for the PhD degree. At the same time he was working in Fisher's lab experimenting on mice, a requirement by Fisher to be his thesis advisor. This was yet another extraordinarily fruitful time for CRR. His research with the University Museum led to major contributions to the field of multivariate analysis, including a read (Discussion) paper at the Royal Statistical Society, two short notes in the journal Nature on biological classification, and an application of his methods to psychometrics. His studies with Fisher stimulated the general score statistics theory mentioned above, and led to a fine paper in the journal Heredity on three-factor linkage analysis in mice, while he continued to contribute to design of experiments and linear model theory. Two themes from this period were to be more fully developed in later papers, and summarized and illustrated in his highly successful books *Advanced Statistical Methods in Biometric Research* (1952) and *Linear Statistical Inference and Its Applications* (1965, 1973). His Indian anthropometric work was published in 1949, while his Cambridge work was published 1955. By the time he left Cambridge, CRR had made a number of seminal contributions to statistics, and with hindsight we recognize him as one the leading statisticians in the world. However, at that time his contributions were not fully recognized, and it took perhaps a decade or more for the statistical community to recognize his achievements fully.

Shortly after his return to Calcutta in 1948, CRR was appointed Professor and head of the Research and Training School at the ISI. This was the beginning of his 30-year role as a leader of the ISI, culminating in his succeeding Mahalanobis as Director-Secretary in 1972. He strengthened the international reputation of the ISI, and he encouraged and supervised 51 research students, many of whom had outstanding research careers in their own right (and according to the Mathematics Genealogy Project, he has 671 academic descendants).

CRR's scientific career continued unabated, with research on growth curves, variance components, second-order efficiency, generalized inverses, and much else — including a popular book Statistics and Truth: Putting Chance to Work. His achievements have been recognized throughout the world through prizes, medals, memberships of learned academies and honorary degrees (see below).

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C.R. Rao: Selected Awards and Achievements

Padma Vibhushan, the Government of India's second highest civilian award
Swarup Bhatnagar Award, Council of Scientific and Industrial Research, India
National Medal of Science, USA
India Science Award
ISI Mahalanobis Prize; ISI President
Fellow of the Royal Society
IMS Wald Lecturer; IMS President
Awarded 39 honorary doctoral degrees from universities in 19 countries around the world
Author of over 475 papers and 15 textbooks
Samuel S. Wilks Memorial Award, ASA
Life Fellow of Kings College, Cambridge
Royal Statistical Society's Guy Medal in Gold and in Silver

Our birthday messages and tributes to CRR

Anirban DasGupta has collected together birthday messages and tributes to CRR from over 30 leading statisticians from around the world, whose lives and work has been touched by him. Read tributes here from G. Jogesh Babu, Anil K. Bera, Jim Berger, Rabi Bhattacharya, Mary Ellen Bock, Florentina Bunea, Herman Chernoff, Anirban DasGupta, Morris Eaton, Piet Groeneboom, David J. Hand, David R. Hunter, Ildar Ibragimov, Iain Johnstone, Hira Koul, Soumendra Lahiri, Runze Li, Wei-Liem Loh, Amita and Partha Majumder, Susan Murphy, Vijay Nair, Michael Perlman, Dominique Picard, Dimitris Politis, B.L.S. Prakasa Rao, Indrani Saha, Bimal Sinha, Kalyan Sinha, Terry Speed, Michael Stein, Bill Strawderman, Sara van de Geer, S.R.S. Varadhan, Brani Vidakovic and Larry Wasserman:

Professor Rao's prodigious work *Linear Statistical Inference and Its Applications*, published more than half a century ago, is still a valuable resource, and relevant. I am glad that I could help bring you to Penn State where your presence greatly enhanced the reputation of the department. Thank you Professor Rao for being a great collaborator, colleague, and mentor. Thank you also for giving a well received presentation on 'model selection' to astronomers during the first three years of our Center for Astrostatistics' annual summer schools on Statistics for Astronomers.

— G. Jogesh Babu

It is my distinct pleasure to send you greetings from your friends and colleagues at the International Statistical Institute. We want to wish you a very happy birthday. According to our membership database, you became an ISI elected member on 01 January 1953, and we are pleased to share this long history of connection with you. Your foundational contributions to the discipline of statistics continue to be celebrated as is your leadership of statistical societies including the International Statistical Institute. Your career spanned work in India, the United States and other parts of the globe, and we are honored that you contributed to the leadership of the International Statistical Institute. Thank you again for your many contributions, and best wishes on your birthday!

— A. John Bailer

1920–2020: the wonderful CR Rao century in statistics, and more to come.

— Anil K. Bera

Everyone knows that CR Rao is one of the greatest statisticians who ever lived. But what I most remember is a man who was incredibly kind and supportive as I grew up in the profession. He is not an avowed Bayesian, but his random effects analysis at multi levels of modeling paralleled and completely complemented Bayesian hierarchical modeling. I treasure all of my interactions with him.

— Jim Berger

My heartfelt congratulations on this marvelous milestone in your glorious life as one of the greatest ever statisticians. Aside from your remarkable achievements, statisticians of all generations will always revere

Professor Mahalanobis and you as the two main architects of the Indian Statistical Institute, a leading center of learning in the world. To us, and our children, your name will always shine as a beacon of inspiration.

— Rabi Bhattacharya

Happy 100th birthday to an International Treasure! In this age of uncertainty, we are especially grateful for your contributions to our understanding of the limitations of describing the probability densities that give the logic of our uncertainty. A well-remembered example of your generosity to your colleagues and students was your 1988 presentation of the first memorial lecture at Purdue University in honor of K.C.S. Pillai.

— Mary Ellen Bock

It is an honor and a privilege to congratulate Professor Rao on his centennial birthday. Unbeknownst to him, he played a major role in my development. I happened upon his book 'Linear Statistical Inference and Its Applications' almost by accident, as I was considering a PhD in functional analysis, and was looking for possible applications. I read this text in one go, and shortly after that I decided to continue my studies in statistics. The clarity of Dr. Rao's writing, and his passion for transmitting knowledge have been an inspiration for me over the years. I am mdeeply grateful for his contributions to our field, and it is with immense joy that I wish him, in 2020, many happy returns.

— Florentina Bunea

To CR, I am still trying to follow in your footsteps. It is a pleasure to know you and to see how it is possible to keep active as the field changes. You and your work has been a guiding light for me and many of us junior to you.

— Herman Chernoff

Dr. Rao walked randomly into my first year descriptive statistics class at the ISI and gave us a lecture on categorical data. At the end of the lecture, one of us asked him what is his name. We were stunned when he wrote 'C. R. Rao' in very small letters on the chalkboard. I got to know him better after I came to the US. I understood some of his work a lot better after I started to teach inference, multivariate analysis, advanced testing, advanced linear algebra, regression theory, and probability. I do not have the audacity to judge his work. On a purely personal level, my top twenty five personal favorites from his mountain of work are the papers on Cramer-Rao inequality, the score test, Rao-Blackwell theorem, invention of orthogonal arrays, growth curves, initiation of higher order efficiency, invention of MINQUE theory, inference for singular regression, the Rao-Shinozaki work on estimators for linear models, the Rao geodesic metric paper on ecology, the J. B. S. Haldane Centenary Lecture, the work with Ranajit Chakraborty on population genetics, the survey paper on statistical models in economics, the papers on statistical methods in psychiatry, the very early work in India and Africa on anthropometry and archaeology, the Rao-Rubin theorem, and the papers on the Cauchy-Deny functional equations, the paper on de Finetti's theorem in damage models, the work on bootstrap in survey sampling, the paper(s) on multiple constantly moving targets, the work on reconstruction of images from two projections, the work on estimation of number and directions of signals when superimposed, and the three classic books, Linear Statistical Inference, Theory of Generalized Inverses, and Characterization Problems in Mathematical Statistics, and the popular book Statistics and Truth. The chapter on measure theory in Linear Statistical Inference can be used even today for a flawless first course on measure theory to PhD students, and the chapter on vector spaces and matrices for a solid course on linear algebra. I want to also mention the beautifully written 1952 biometry

book, and especially the delightful exercises at the end. Herman Rubin once remarked to me that C. R. Rao is among the very few best statisticians in history. There will never be another C. R. Rao in the unimaginable range of contributions to science as a whole, and the lasting impact that the work has had across disciplines. He is a unique phenomenon. Random things shape one's life, someone said. What would have happened to statistics if C.R. Rao did not happen to meet P.C. Mahalanobis? Happy Birthday, Dr. Rao. You have made me and every Indian proud.

— Anirban DasGupta

Congratulations on your hundredth birthday and your illustrious career.

— Morris Eaton

The book "Linear Statistical Inference and its Applications" is the first book that made me understand the theory of statistics. I feel extremely grateful for it and his work and I wish him a very happy 100th birthday.

— Piet Groeneboom

Dear Professor Rao, Happy Birthday. Congratulations on this latest achievement, even rarer than all the gold medals.

— David J. Hand

Happy Birthday Professor Rao and thank you for all the wonderful memories! From a former colleague at Penn State who still delights in talking about the Rao score test and the Cramer-Rao inequality, and proudly talking about your connection to Penn State, class after class of graduating students.

— David R Hunter

Dear Professor Rao, I was happy to know personally some creators of our science: Cramer, Kolmogorov, Linnik, and YOU. I remember Linnik's lectures on mathematical statistics in which he told us of the famous Cramer-Rao inequality and equally famous Rao-Blackwell sufficient statistics improvement. Happy birthday to you, dear Professor Rao!

— Ildar Ibragimov.

Congratulations to an icon of our field on yet another remarkable achievement.

— Iain Johnstone

It was an honor for me to meet Dr. Rao in 1965 at Berkeley. He was invited to the fifth Berkeley Symposium. Ever since that first day, he has been a great inspiration personally and professionally, as he has been to

numerous others in the field. Dr. Rao is also a gentleman, and so kind. I wish him a very happy 100th birthday.

— Hira Koul

There is hardly any area of statistics that has not been influenced by Dr. Rao's work. From the information inequality to the Rao-Blackwell sufficiency reduction to uniform asymptotic efficiency to hands on multivariate analysis and experimental design, survey sampling and quality control, and biology and ecology, Professor Rao has made profound contributions and in many cases laid the foundations across these diverse areas. His results will continue to serve as the basic guiding principles in statistics for years to come. Happy Birthday.

— Soumendra Lahiri

Professor Rao, I am blessed to have a great colleague and friend like you. I would like to take this opportunity to wish you the best birthday this year. May your special day be filled with happiness.

— Runze Li

A very happy 100th birthday to Professor Rao. In the field of statistics, he is one of the giants of the twentieth century. Many years ago, Dr. Rao visited Singapore and we had a dinner in his honor. At that dinner, I realized what a witty person he is and that he has a beautiful sense of humor.

— Wei-Liem Loh

When I first joined the ISI, our classes used to be held at the famous Room 3.0 on the 3rd floor of the main building. We would see an unassuming gentleman walk out of the elevator and enter his office. Over many years, when we got to know him and understand some of his contributions, we felt he is one of the most intelligent people we have ever met. And, a simple man with a great sense of humor. That's Professor CR Rao.

— Amita and Partha Majumder

Happy Birthday Professor Rao! I have always admired you and all your work. I still remember that when I interviewed at Penn state, I made them change my entire schedule just so I could meet with you. It is an honor that I spoke with you.

— Susan Murphy

Dear Dr. Rao, It is a privilege to wish you a happy birthday. You have had an indelible mark on the fields of statistics and Data Science through fundamental discoveries in so many areas. Your Linear Statistical Inference book was our bible in graduate school. Thank you also for all of your contributions to statistics in India, to the Indian Statistical Institute, and for your leadership at the "other ISI" and numerous statistical societies and Sankhya.

— Vijay Nair

I am extremely happy for this opportunity to congratulate you on your centennial anniversary. Along with countless other aspiring students of statistical science, I have profited greatly from your many invaluable contributions to our discipline. In particular, my copy of your textbook Linear Statistical Inference is filled with copious notes and comments that I added during many hours of studying your treatment. Please accept my warmest thanks for your exemplary leadership in our field.

— Michael Perlman

Congratulations on your 100th birthday. You are such a brilliant scientist. You have been my hero for years and a role model for my own work all my professional life. I do not have enough words to express my gratitude for your inspirational work.

— Dominique Picard

Warmest wishes to a gentle giant of our field. What a remarkable career.

— Dimitris Politis

Contributions by Professor Rao to the field of statistics are well-documented, and I do not need to repeat them. I do want to mention his very significant contributions to research and education within India, and his consistent encouragement of excellent work in every area of mathematics, statistics, and their applications.

— B.L.S. Prakasa Rao

Dr. Rao, you are a great teacher, expositor and statistician of international stature. Your text ‘Linear Statistical Inference and Its Applications’ is an invaluable collection in the treasure trove of numerous students and researchers in statistics and mathematics. You will remain a beacon to statisticians and researchers in many fields of the sciences for all time.

— Indrani Saha

Professor Rao, on this divine occasion of your centenary birthday, I offer my humble gratitude. During the sixties and the seventies, as an undergraduate, Masters and PhD student at the University of Calcutta, we all grew up with your many fundamental and novel contributions. I was blessed too to be your colleague at the University of Pittsburgh. I will forever remember my academic and social interactions with you.

— Bimal Sinha

When I joined the Delhi Centre of the ISI in 1979, Professor Rao was away and left for the United States soon after. I had heard a lot of anecdotes about the legendary Dr. Rao. It is not really for me to talk about Dr. Rao’s innumerable path-breaking discoveries in statistics. Possibly a sense of curiosity as well as the thinking of links with other areas of mathematics that brought me in touch with some of Dr. Rao’s ideas. For example, the

Cramer-Rao inequality, generalized inverses of singular matrices, and the Riemannian differential metric on probability distributions. Later during my stint as the director of the ISI, I had close interactions with Dr. Rao. The first time was in 2000, when his 80th birthday was celebrated, and I had the privilege to release the fifth volume of his collected papers, and the second time was in 2003, when a special function was held at the Calcutta ISI to celebrate his seminal contributions in statistics, as also his towering role in making the ISI the great institution that it is today. I am honored to offer my tributes to Dr. Rao for everything, and for what he has done for statistics in India and for the ISI.

— Kalyan Sinha

Greetings and best wishes to CR Rao on the occasion of his 100th birthday, from an admirer of nearly sixty years.

— Terry Speed

I made heavy use of Dr. Rao's book Linear Statistical Inference and Its Applications during phases of my career. The first chapter, "Algebra of Vectors and Matrices" is most valuable. It provides a succinct and well-chosen set of results from linear algebra for the statistical researcher. This book is still very worth owning nearly 50 years after the publication of the second edition.

— Michael Stein

Dr. Rao is obviously in the pantheon of 20th century statisticians. His contributions to statistical theory and methodology are among the deepest, broadest and most consequential in history. His contributions to statistical education through his excellent texts, his teaching, and his leadership are all of immense significance too. Congratulations, Dr. Rao, and best wishes.

— Bill Strawderman

Dear Professor Rao, thank you for all your work that spread over the world and made the subject of statistics flourish. We all collect the fruits of your insight from our statistical garden. This harvest of generations of statisticians is due to you standing at the root and planting the precious seeds. Congratulations.

— Sara van de Geer

CR Rao is one of the greatest statisticians of all time in the tradition of Pearson and Fisher. And, he built up an international center of excellence in theoretical and applied statistics and mathematics during his time at the RTS in the ISI.

— S.R.S. Varadhan

You have been a blessing to the statistical community of the whole world. With admiration, honor, and a salute, we celebrate your 100th birthday.

— *Brani Vidakovic*

Dr. Rao, your ideas have helped shape the entire field of Statistics. It is hard to overstate how influential your work has been. And, beyond statistics too. I just saw an announcement of your birthday in a group of astrophysicists. Happy Birthday.

— *Larry Wasserman*

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